1 Completing The Square - Example 1

For our first example, we'll examine a simple quadratic.

Let $f(x) = x^2 + 2x + 4$. We will complete the square here using only the axis of symmetry and a little planning.

When we complete the square, we want to find an equation of the form $ax^2 + bx + c = a(x - h)^2 + k$, where h and k are the numbers to be found — h is the same number we get when we find the axis of symmetry, and k is the value of the quadratic at $x = h$.

Recall that the axis of symmetry for any quadratic lies at $\frac{b}{2a}$: here $a = 1$ and $b = 2$, so the axis of symmetry lies at -1. Putting this in the 'square' part of our equation, we get $x^2 + 2x + 4 = (x - (-1))^2 + k$, which we can rewrite as $(x + 1)^2 + k$.

We must now find k - although we could find the value of the function at $x = -1$, for diversity's sake we'll find it directly.

Expanding the equality we found with the axis of symmetry, we have $x^2 + 2x + 4 = x^2 + 2x + 1 + k$; subtracting like terms from each side, we get $k = 3$.

We then have $x^2 + 2x + 4 = (x + 1)^2 + 3$.

2 Completing The Square - Example 2

In this example we'll use the vertex directly.

As mentioned in the class and document, when we find the vertex—which we will call (h, k) for convenience—we can insert h and k directly into the formula $ax^2 + bx + c = a(x - h)^2 + k$ and be done. For this reason we refer to $a(x - h)^2 + k$ as the *vertex form* of a quadratic.

Suppose we have $f(x) = x^2 - 6x + 12$. We can find h easily using the formula for the axis of symmetry.

$$\begin{aligned} h &= \frac{-b}{2a} \\ &= -\frac{-6}{2} \\ &= \frac{6}{2} \\ &= 3. \end{aligned}$$

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We can then find $k = f(h)$ by plugging this into our function.

$$\begin{aligned} k &= f(h) \\ &= f(3) \\ &= (3)^2 - 6(3) + 12 \\ &= 9 - 18 + 12 \\ &= 3 \end{aligned}$$

Putting these values for h and k into the vertex form equation given above, we have $x^2 - 6x + 12 = (x - 3)^2 + 3$.