Homework - MAT 1214.01F

Due 8 June 2009

2 June

Display a function (not already used as an example in class) that is

1. continuous everywhere;

2. discontinuous at exactly one point;

3. discontinuous at the members of a finite collection $\{x_1, \ldots, x_n\}$ of real numbers and nowhere else;

4. discontinuous everywhere.

3 June

Display a continuous function (not already used as an example in class) that is

1. differentiable everywhere;

2. differentiable everywhere but one point;

3. differentiable everywhere except at the members of a finite collection $\{x_1, \ldots, x_n\}$ of real numbers.

4 June

1. Let $p(x) = (x - a)^2$. Is a a root of $p'(x)$?

2. Let $q(x) = (x - a)^3$. Is a a root of $q'(x)$? $q''(x)$?
   (Note: $q''(x)$ is the second derivative — the derivative of the derivative — of $q(x)$. Differentiate $q(x)$ to get $q'(x)$, and then differentiate $q'(x)$ to get $q''(x)$.)

Due 15 June 2009

8 June

Discuss the asymptotic behaviour of

$$f(x) = \frac{x^3 + 3x^2 + 2x + 1}{x}.$$

9 June

1. Discuss the asymptotic behaviour of
$$f(x) = \frac{x^8}{x^4 - 1}.$$
2. Find the derivative of $f(x) = sin\left(\frac{1}{sin(x)+3}\right)$.
3. Find the derivative of $f(x) = e^{sin(x)+2}$
4. Bonus: Let n be a fixed integer greater than 0. Let x_0 be a fixed real number. Find the line tangent to $f(x) = x^n$ at x_0.

10 June

For which integers n is $f(x) = |sin^n(x)|$ differentiable?

11 June

Don't get hit by a bus the day before the test. That would be silly.

12 June

Find a function continuous only at numbers of the form $\pm n^2\pi$ ($\pm$ square integer multiples of pi) defined on all real numbers.

Due 22 June 2009

16 June

1. Let $f(x) = 3x + 2, g(x) = 6x + 9$. Find $f^{-1}(x), g^{-1}(x), g \circ f(x), (g \circ f)^{-1}(x)$. (Note: here $f^{-1}(x)$ means 'the inverse of f'.)
2. Do the same with $f(x) = ax + b, g(x) = cx + d$ where a, b, c, d are arbitrary, fixed real numbers and $a, c \neq 0$.

18 June

Find, if they exist, the absolute and local maxima and minima of

1. $x^3 - 3x + 4$,
2. $(sin(x^2))^2$,
3. e^x.

19 June

1. When can a polynomial (on the whole real line) have both a global maximum **and** a global minimum? Only a global maximum or a global minimum? No global maxima or global minima?
2. How many maxima and minima (global or local) can a polynomial (on the whole real line) of degree n have? (Hint: Fundamental Theorem of Algebra.)

Due 29 June 2009

22 June

Prepare for the exam Wednesday by reviewing what has been done and discussed in class. (Suggestion: Understand, and be able to use, the quadratic formula!)

23 June

Know the quadratic formula. Really. It's good for you.

24 June

Let $a, b, c > 0$. Find the point (x_0, y_0) on the curve $ax^2 + by^2 = c$ such that $x_0, y_0 > 0$ and $\frac{dy}{dx} |_{(x_0,y_0)} = -1$.

25 June

1. Find and classify the extrema of $f(x) = 2x^3 + 3ax^2 + 6bx + 6c$.
2. Find $\lim_{x\to 0} \frac{sin(x)}{x}$.
3. Find $\lim_{x\to 0} \frac{x}{sin(x)}$.

26 June

Compute

$$\lim_{x\to 1} \frac{x^n - 1}{x - 1}$$

using

1. Factorization of the rational expression $\frac{x^n-1}{x-1}$,
2. L'Hopital's rule

Bonus

Using basic geometric reasoning, show that

1. The area under the graph of $f(x) = ax$ between 0 and 1 is $\frac{a}{2}$,
2. The area under the graph of $f(x) = ax$ between 0 and any positive real number t is $\frac{at^2}{2}$.

Due 1 July

29 June

Compute $\int \frac{x^3}{1+x^4} dx$.

30 June

Compute $\int \tan x dx$.