INTRODUCTION

The Texas Differential Equations Conference is an annual event that brings together researchers working on differential equations and related topics. The Texas DE was first organized by Professors Charles Radin and Ralph Showalter at the University of Texas at Austin in 1978, originally as Texas PDE Seminar. The Texas DE Conference has a strong democratic tradition: There are no principal speakers, all talks are allocated the same length of time, and there is no registration fee. The intent is to encourage participation by young faculty and graduate students.

The conference is supported by the American Institute of Mathematical Sciences, the UTSA College of Sciences, and the UTSA Department of Mathematics.

This years conference was organized by Fengxin Chen, Mostafa Fazly, Changfeng Gui, and the staff of the Department of Mathematics.
NEW CONJECTURES FOR THE HIGGS BOSON EQUATION IN THE DE SITTER SPACETIME

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Based on high performance computations we show some new conjectures for the Higgs boson equation in the de Sitter spacetime. There are several interesting open questions about this semilinear Klein-Gordon equation in the de Sitter space-time. Other than the existence and long time behavior of solutions, the existence and interaction of bubbles formed by zero isosurfaces with sign change is also of interest. Our numerical simulations suggests that 1. solutions converge to step functions related to unforced, damped Duffing equations. 2. smooth solution exists for all time if the initial conditions are compactly supported and smooth; 3. under some conditions no bubbles form.

CONVECTIVE FLOW IN A POROUS MEDIUM WITH LINEAR VERTICAL VARIATION IN RESISTIVITY

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We consider a horizontal porous medium with rigid top and bottom boundaries to analyze hydro-thermal convective flow. The governing system consists of continuity equation for conservation of mass, heat equation for conservation of energy, and conservation of momentum equation which is governed by Darcy’s law. We investigate the effect of linear variation of resistivity in the vertical direction. Assuming a vertically varying basic state, we apply weakly nonlinear approach to compute the solutions for quantities such as temperature and vertical velocity. Numerical results for those quantities are presented.
APPLICATION OF STOCHASTIC DIFFERENTIAL EQUATION BY USING ORNSTEIN UHLENBECK PROCESS

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This work is devoted to the study of a stochastic differential equation. We propose a differential equation arising from the superposition of independent Ornstein–Uhlenbeck processes driven by Gamma (a, b) process. The superposition of independent Gamma (a, b) Ornstein–Uhlenbeck processes offers analytic flexibility and provides a class of continuous time processes capable of exhibiting long memory behavior. The stochastic differential equation is applied to finance by fitting the superposed (a, b) Ornstein–Uhlenbeck model to typical financial time series. We see that the Ornstein Uhlenbeck process followed in this work simulates well the financial time series, which aligns our estimation algorithm with long memory behavior. The presence of long memory suggests that current information is highly correlated with past information at different levels, what may facilitate parameter estimation and prediction for financial data.

EXTENSION PROBLEM AND HARNACK INEQUALITIES FOR MASTER EQUATIONS

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We introduce the fractional powers of parabolic operators in divergence form. These equations show up in continuous time random walk models as the master equation. Some other examples include the parabolic Signorini problem and the phenomenon of osmosis. We study these nonlocal problems by a localization procedure. Indeed, by using the method of semigroups we can prove a parabolic extension problem. As the theory of this local problem is well established, we use it to prove Harnack inequalities for the nonlocal problem. We present parabolic Harnack estimates, both in the interior and up to the boundary.
FRACTURE MECHANICS IN DIFFERENT ELASTICITY THEORIES AND A HIERARCHY OF PDES

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Different types of materials are considered; they include homogeneous versus nonhomogeneous, linear and nonlinear, and materials with size-effect. Different elasticity theories are used to describe various types of material behavior, and they lead to a hierarchy of governing PDEs in terms of displacement. Surprisingly, there is one unified way to test the discussed elasticity theories.

MULTIDIMENSIONAL STABILITY OF PLANAR TRAVELING WAVES FOR NONLOCAL ALLEN-CAHN EQUATION

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It is well known that the traveling wave for nonlocal Allen-Cahn equation is exponentially orbitally stable in one dimensional bistable case. The main concern of this paper is the multidimensional stability of planar traveling waves. It is shown that it is stable for $x \in \mathbb{R}^n$ for $n > 2$. Furthermore, it is shown that the perturbation of the wave decay at the rate of $\varepsilon^{(n+1)/2}$. 
EXISTENCE OF A UNIQUE CLASSICAL SOLUTION TO A QUASILINEAR ELLIPTIC EQUATION

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We consider a quasilinear elliptic equation under a Neumann boundary condition. The value of the solution at a given point in the domain is known. We prove the existence of a unique classical solution. The key to the proof lies in obtaining a priori estimates for the solution.

EXISTENCE OF SOLUTIONS TO THE UPERCRITICAL HARDY-LITTLEWOOD-SOBOLEV SYSTEMS

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It is known that for the supercritical Hardy-Littlewood-Sobolev (HLS) systems with integer-power Laplacians, there exist classic solutions. This is proved by a shooting method. For HLS systems with fractional Laplacians, there also exist classic solutions, and this can be proved by a bifurcation method. In this talk, the speaker will describe the strength and limitation of both methods. Also, involving fractional Laplacian, the speaker will discuss the regularity of solutions to the Dirichlet problem and an integration-by-part formula.
NONSTANDARD DISCRETIZATION METHODS OF DIFFERENTIAL EQUATIONS
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In this talk we will propose a nonstandard discretization scheme that preserves all the dynamics, including bifurcation and stability, of the differential equation. The method is particularly useful for biological and economical applications, when the step size or (census time, data collection time) is not small. Examples include one-, two and three dimensional system of differential equations. Though PDE is not included in the talk, but the method applies to PDE as well.

LOCATING DEFECTS IN A VIBRATING STRING USING EIGENFUNCTION EXPANSION AND THE METHOD OF MOMENTS
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In this talk, we consider a vibrating string of length one with uniform density except at some defects. We tackle the problem of locating these defects and possibly determining their sizes by using some measurements of the vibrations in the string after the application of an impulse near one of the endpoints. Our analysis relies mainly on the eigenfunction expansion of the solution of the partial differential equation characterizing the vibrations in the string and on the method of moments. Numerical results supporting this analysis will be presented as well.
ON DE GIORGI’S CONJECTURE IN LOWER DIMENSIONS

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We begin this talk with an introduction on fundamental ideas behind the De Giorgi’s conjecture (1978) that is inspired by the Bernstein’s problem (1910). This conjecture brings together three groups of mathematicians: (a) a group specializing in nonlinear partial differential equations, (b) a group in differential geometry, and more specially on minimal surfaces and constant mean curvature surfaces, and (c) a group in mathematical physics on phase transitions. We then present a few natural generalizations of the conjecture. These generalizations lead us to introduce certain novel concepts, and we illustrate why these novel concepts seem to be the right concepts in the context and how they can be used to study particular systems and models arising in Mathematical Physics.

BILINEARIZATION AND REDUCTIONS FOR THE NLS, DERIVATIVE NLS, COMPLEX SINE-GORDON AND FOKAS-LENELLS EQUATIONS

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In this talk, we will first present the bilinear equations for the nonlinear Schrodinger (NLS), derivative NLS, complex sine-Gordon (csG) and Fokas–Lenells (FL) equations. Then, starting from tau-functions of a two-component Kadomtsev–Petviashvili (KP)-Toda hierarchy and a set of bilinear equations satisfied, we will show that the multi-soliton solutions to all above equations are reduced from the same class of tau functions.
Abstracts

FINITE LIFESPAN OF SOLUTIONS OF THE SEMILINEAR WAVE EQUATION IN THE EINSTEIN-DE SITTER SPACETIME
Anahit Galstyan, The University of Texas at Rio Grande Valley
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We present some results on the semilinear massless waves propagating in the Einstein-de Sitter spacetime. We examine the solutions of the semilinear wave equation, and, in particular, of the \( \varphi^p \) model of quantum field theory in the curved space-time. More precisely, for \( 1 < p < 4 \) we prove that solution of the massless self-interacting scalar field equation in the Einstein-de-Sitter universe has finite lifespan.

IMPLICIT SOLUTIONS TO THE KDV-BURGERS-KURAMOTO EQUATION
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In this talk, we consider the KdV-Burgers-Kuramoto equation, a partial differential equation that occupies a prominent position in describing some physical processes in motion of turbulence and other unstable process systems. Equivalence transformations are applied for exploring the principal Lie algebra. By means of the associated equivalence algebra we convert partial differential equations into a system of ordinary differential equations from which we can derive some explicit and implicit solutions.
MODELING OF THE TRACHEA OPENING SIZE CHANGE UNDER ANGIOEDEMA
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Tracheal angioedema is a pathology of the airway caused by soft tissue swelling due to fluid leakage from the blood vessels. This pathology can suddenly change the normal tracheal luminal size and cause breathing difficulty for an medical emergency. The extra fluid accumulation inside the tissue can also alter the stiffness of the tissue, and make the luminal size change more complicated. We set up a model using continuum mechanics to understand how the angioedema swelling extent can quantitatively change the trachea luminal size particularly under the tissue stiffness modification. Interestingly, the swelling may not always shrink the tracheal lumen, and instead may expand it for proper parameter values. This model can assist conducting more appropriate medical treatment for tracheal angioedema.

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QUANTITATIVE BOUNDS VERSUS WEAKLY COUPLED STATES FOR SOME PSEUDORELATIVISTIC OPERATORS
Vu Hoang, The University of Texas at San Antonio
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(joint work with D. Hundertmark, J. Richter and S. Vugalter) In this talk, I discuss the question of existence of weakly coupled bound states for a wide class of Schroedinger-like operators. These operators have applications to superconductivity and relativistic one-body and two-body problems. In particular, I present a sharp condition on the kinetic energy part of the operator that characterizes the existence of weakly coupled bound states.
BLOW-UP SOLUTIONS FOR A MEAN FIELD EQUATION ON SPHERE AND TORUS

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We consider the mean field equation

$$\Delta_g u + \rho \left( \frac{h e^u}{\int_M h e^u} - \frac{1}{|M|} \right) = 0$$

where $h$ is a positive function, $(M, g)$ is a closed Riemann surface, $\Delta_g$ is the associated Laplace-Beltrami operation and $|M|$ is the total area of the surface. It is well known that nondegeneracy condition on a combination function $\int_M h$, the curvature and the Green function implies the existence of blow-up solutions as $\rho \to 8\pi m$ when $m$ is any positive integer. We construct blow-up solutions of the degenrate case when the underlying manifold $M$ is a sphere or torus and $h \equiv 1$ by assuming some additional symmetry. This is a join work with Ze Cheng (UTSA) and Changfeng Gui (UTSA).

OPTIMIZATION OF DIFFERENTIAL EQUATIONS FOR MOONSHINE FUNCTIONS

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We construct a weight 2 modular form $z$ from a Hauptmodul $x$ for certain congruence subgroups called moonshine groups. The function $z$ satisfies a linear third order differential equation with respect to $x$. This differential equation will be described explicitly from the construction, and techniques for minimizing the degree of the coefficients will be discussed. Singular values of $x$ and corresponding series expansions for $z$ in terms of $x$ allow for the construction of rapidly convergent series expansions for $1/\pi$ that generalize the work of Ramanujan and others.
A PROOF OF ONSAGER’S CONJECTURE

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I will discuss a proof of Onsager’s conjecture that 1/3-Hölder is the threshold regularity for energy conservation for the 3D incompressible Euler equations, a conjecture motivated by theories of anomalous dissipation and energy cascades in turbulent fluid flow. The construction is based on the method of convex integration, which was first developed for the incompressible Euler equations by De Lellis and Székelyhidi to build Hölder-continuous Euler flows that fail to conserve energy, and was later improved by Isett and by Buckmaster-De Lellis-Székelyhidi to obtain further partial results towards Onsager’s conjecture. The proof to be discussed of the full conjecture combines the use of “Mikado flows” due to Daneri-Székelyhidi with a new “gluing approximation” technique. The latter technique exploits a special structure in the incompressible Euler equations and Euler-Reynolds equations.

MIXED BOUNDARY VALUE PROBLEMS FOR NON-DIVERGENCE TYPE ELLIPTIC EQUATIONS IN UNBOUNDED DOMAIN

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We investigate the qualitative properties of solution to the Zaremba type problem in unbounded domain for the non-divergence elliptic equation with possible degeneration at infinity. The main result is Phragmén-Lindelöf type principle on growth/decay of a solution at infinity depending on both the structure of the Neumann portion of the boundary and the “thickness” of its Dirichlet portion. The result is formulated in terms of so-called s-capacity of the Dirichlet portion of the boundary, while the Neumann boundary should satisfy certain “admissibility” condition in the sequence of layers converging to infinity.
LAX REPRESENTATION FOR AN INTEGRABLE HIERARCHY

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Using the functional gradient approach of eigenvalues, this talk presents a pair of Lenard’s operators for the Levi’s vector fields and establishes commutator representations for hierarchies of Levi’s equations. The relationship between potential and stationary Levi’s system is discussed in the end.

EPIDEMIC MODELING AND IDENTIFYING DOMINANT TRANSMISSION PATHWAYS

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Emerging infectious diseases are a threat to biodiversity and fungal pathogens have caused rapid declines in amphibian populations around the globe (McCraw and Gurr, 2012). Gray et al. (2015) identifies Batrachochytrium salamandrivorans (Bsal) as an emerging fungal pathogen that caused rapid die-offs of naïve salamanders in Europe and predicts North America will soon experience similar devastation if no policy actions are taken and the pathogen emerges. Epidemic dynamics of infectious diseases with multiple routes of transmission are complex. Mathematical models can be used to determine invasion potential and identify which transmission pathway is dominant, can ultimately help identify appropriate intervention strategies. We developed compartmental host-pathogen models to examine transmission dynamics of an emerging fungal pathogen on an amphibian population. Multiple stages of infection are incorporated into the model, allowing disease-induced mortality and zoospore shedding rates to increase as the disease progresses. Parameter sensitivity analysis using the Latin Hypercube Sampling (LHS) with the Partial Rank Correlation Coefficient (PRCC), calculations of the basic reproductive number, and numerical simulations shed insight into pathogen dynamics.
INTERACTION AMONG INFINITE NUMBER DISLOCATIONS AND A SEMI-INFINITE CRACK IN ONE-DIMENSIONAL HEXAGONAL PIEZOELECTRIC QUASICRYSTALS

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Quasicrystals (QCs) are a novel structure of solids discovered by Shechtman et al. in 1984, which are orientationally ordered, but with noncrystallo-graphic. Due to this discovery, Shechtman won the 2011 Nobel prize for chemistry. Since the quasicrystal was found, generalized elasticity theory of quasicrystals has been formulated in terms of partial differential equations, and many analytical solutions for defect problems of QCs are obtained. As far as the problems of finitely many defects in QCs are concerned, there are some research papers. But on the interaction among infinitely many defects in QCs, few results have been reported. For piezoelectric QCs, these problems become more complicated than conventional material owing to the introduction of phason field and electric field. The purpose of this paper is to consider the interaction of more defects in one-dimensional hexagonal piezoelectric QCs with partial differential equation theory and complex variable function method. First, the analytic solutions of elastic field of dislocation in the material are studied in forms of complex variable function; then the interaction force of parallel dislocations are given. Second, the analytic solutions of elastic field of semi-infinite crack in the material are obtained by using Muskhelishvili method. Finally, the elastic fields of interaction of among infinitely many parallel dislocations and a semi-infinite crack are given, which offer the basis of theory to discuss dislocation emission in the top of crack, dislocation screen and crack shielding in one-dimensional piezoelectric hexagonal. Meanwhile, these theories are the development of the corresponding parts of classical elastic theory to piezoelectric QCs.

THE EXISTENCE OF NONTRIVIAL SOLUTIONS FOR SEVERAL KINDS OF ELLIPTIC SYSTEMS ON R^N

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The study on the existence of nontrivial solutions of elliptic systems has been an important research field in nonlinear Functional analysis and differential equations. By using the variational methods and the critical point theory, in this talk, we will discuss the existence of solutions for several kinds of elliptic systems on R^N.
DERIVATION OF THE GENERAL ENTRepic Function
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Entropy is one of the basic and most important macroscopic physical quantities in thermodynamics. The property of composability states that the entropy of the total system, composed of any two independent subsystems A and B, is a function of the entropies of the subsystems, \( S = F(S_a, S_b) \); this function \( F \) constitutes the additivity rule of entropy. Starting from the zero-th law of thermodynamics, and using the composability property and Schwarz’s integrability condition, we derive three sets of partial differential equations. The first two sets define the two independent intensive parameters, the temperature and kappa index, spanning a 2-D thermodynamic space. The third set can be solved to find the most general form of composability function \( F \). Finally, the composability function is used to derive the most general form of entropy function, \( f \). In particular, given the discrete distribution function \( \{p_1, p_2, p_3, \ldots\} \), associated with particle energies \( \{E_1, E_2, E_3, \ldots\} \), the entropy is given by the summation on all the probabilities, \( S = f(p_1) + f(p_2) + f(p_3) + \ldots \). Starting from the already derived composability function \( F \), we show that the entropic function \( f \) is a solution of the Gaussian hypergeometric differential equation.

WELL-POSEDNESS FOR AN AXISYMMETRIC DIV-CURL SYSTEM
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The div-curl system is a popular model in mathematical physics which asks to reconstruct a vector given its divergence and curl. On bounded domains, we may attach prescribed normal or tangential boundary conditions to the div-curl system. These boundary conditions need to be compatible with the prescribed sources in the interior. Further, the null space of the system with normal or tangential boundary conditions is nontrivial if the domain has holes or handles. In this talk, I present some well-posedness results for an axisymmetric div-curl system with prescribed normal or tangential boundary conditions on a bounded volume of revolution with toroidal holes.
**THE BLOW-UP OF SOLUTIONS OF WAVE MAP EQUATION WITH PERIODIC IN TIME SPEED OF PROPAGATION**

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We give an example of the influence of a time dependent speed of propagation on the global existence of solutions to the initial value problem for wave map equation. More exactly for the arbitrary small smooth initial data we construct blowing up solutions of the wave map with periodic in time coefficients. For the proof we use the Lyapunov-Floquet theory and Borg’s theorem.

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**RESONANCES OF SMALL VOLUME HIGH CONTRAST LINEAR AND NONLINEAR SCATTERERS: ASYMPTOTIC METHODS**

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We consider the asymptotic behavior of the resonances of high contrast small volume scatterers for linear and nonlinear scalar waves. The resonance problem is formulated as a nonlinear eigenvalue problem. We derive asymptotic formulae for linear scatterers and nonlinear scatterers of Kerr type. Our results are illustrated with numerical examples.
CIRCULAR SPECTRUM AND BOUNDED SOLUTIONS OF PARTIAL FUNCTIONAL DIFFERENTIAL EQUATIONS

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In this talk we present an application of the concept of circular spectrum to study the existence of bounded solutions, especially almost periodic solutions of linear partial functional differential equations that depend on time periodically. Our results are stated in terms of spectral properties of the inputs and spectrum of the Poincare operator determined by the equation in the phase space.

VELOCITY-DECOMPOSITION APPROACH FOR OSCILLATORY STOKES AND BRINKMAN EQUATIONS

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The velocity and pressure fields describing transient flows at small Reynolds numbers (Stokes flows) and flows obeying Brinkman models are connected by vector partial differential equations (PDEs). By using the velocity-decomposition approach we present general solutions of the governing PDEs. The geometry dependent vector representations emerge from incompressibility condition and are expressed in terms of just two scalar functions similar to the famous Papkovich-Neuber and Boussinesq-Galerkin solutions type. New formulae connecting our representations and the existing solutions describing unsteady Stokes flow including that of Lamb’s (1932) general infinite series solution will be shown. An important link between cillatory flows and flow through porous media using Brinkman models will be discussed. Indeed, the solutions of boundary value problems in the latter can be obtained in a straightforward fashion, from the results of the former. This simple but surprising analogy will be explained using the properties (mathematical as well as physical) that are shared by the two different models. Our methods and general solutions will be of interest in the computation of multi-particle interactions in transient and Brinkman flows and can be extended to problems in linear elasticity.
EXISTENCE AND UNIQUENESS FOR WEAK SOLUTIONS OF STRONG CROSS DIFFUSION SYSTEMS

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In this talk, we establish the existence and uniqueness results of nonnegative weak solutions for nonlinear strong cross-diffusion systems, including the SKT model and the model of diffusion of red and blue particles. The existence of weak solutions is obtained by maximum principle and the finite difference in time. The methods that we use to show the uniqueness of weak solutions include duality argument and eigenvalue problem approach.

ACTIVE CONTROL OF FIELDS SATISFYING THE HELMHOLTZ EQUATION

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A method will be presented for controlling a space that satisfies the Helmholtz equation with a collection of controllable sources. The most exciting developed scenario is the ability to generate a back propagating pattern in a region of space with an almost non-radiating field. The applications of which include personal audio spots, shielding, covert communication, and active cloaking.
HIGHER ORDER CH MODEL
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In this talk, we study a higher order Camassa-Holm (HOCH) model with peakon solutions. Some explicit single peakons and two-peakon interactional solutions are discussed in details. This is the joint work with Quansheng Liu.

NUMERICAL OPTIMIZATION PROBLEMS IN NUCLEAR REACTOR DESIGN
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We propose a novel approach for a frontier problem in nuclear reactor design by formulating an inverse source problem as a Partial Differential Equation (PDE)-constrained optimization problem with inequality state constraints. Such inequality state constraints are uncommon in the PDE-constrained optimization community. Our objective is to optimize the heat source so that the fuel rod wall temperature does not exceed its critical condition. In addition, the optimized solutions have to satisfy the PDE modeling the reactor physical system. Our preliminary research showed that regularization was not very important, but globalization to ensure convergence and preconditioning the linear systems are required. Since these problems are large, parallelization of the numerical optimization methods is important. Our main goal is to develop a parallel, locally convergent algorithm for solving our nuclear reactor design problem. Also, we will adopt globalization strategies for general numerical optimization for our nuclear reactor design problems and implement the proposed algorithm for use on high-performance computing resources. The expected scientific and technical impacts from this project include solving a frontier problem in nuclear reactor design, promoting more collaboration with nuclear scientists and engineers, and contributing new ideas for solving PDE-constrained optimization problems with inequality state constraints in nuclear reactor designs. The resulting algorithm will be implemented in Toolkit for Advanced Optimization (TAO) and will provide a new capability for solving PDE-constrained optimization problems with state constraints. Our algorithm will be available for use beyond this project and can be applied to other numerical PDE-constrained optimization problems with state constraints.
NUMERICAL OPTIMIZATION PROBLEMS IN NUCLEAR REACTOR DESIGN

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I will present a new method of orthogonal grid generation and investigate certain theoretical aspects of the optimal control approach to the image registration problem. In the first part of the talk, we will present a variational method of orthogonal grid generation which is based on the deformation method and solving Euler-Lagrange equations. Although the concept of grid generation has been studied extensively, the generation of orthogonal grids is still one of the most challenging problems of the grid generation methods. An orthogonal grid should offer significant advantages in the solution of systems of partial differential equations. Previous work requires a uniform grid to improve the orthogonality of grids. The grid deformation method provides size control via a monitor function but it has no control over gridline orthogonality. This approach is to improve orthogonality while providing size distribution by the deformation method. In the second part of the talk, we replace the cost functional in the orthogonality problem with the sum of squared differences known as SSD to solve the Image registration problem. Image registration is a significant part of image processing. It is a process of finding an optimal geometric transformation between corresponding pixels that minimizes the SSD. In this optimal control approach, the grid deformation equations are used as constraints. Existence of optimal solutions is proven using the direct method in the calculus of variations; discuss the non-uniqueness of solutions and the existence of Lagrange multipliers of the optimal control problem for image registration using an abstract theorem concerning the existence of Lagrange multipliers on Banach Spaces.
AN APPROXIMATION OF THE N-SOLITON SOLUTION OF KDV

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The N-soliton solutions of the Korteweg-de Vries equation have been subject of numerous studies. The interest stems from the mathematical and the physical importance of those solutions. Explicit N-soliton solutions can be obtained by using the Hirota’s method or the Wronskian determinant but the physical interpretation is obscured by the complexity of the formulas. Asymptotic analysis was considered by several authors, in particular T. Molony and P.F. Hodnett, to study the interactions of the single solitons building the N-soliton solution. In the talk we discuss an approximation of the multilinear functions of the type

$$\text{det} \begin{pmatrix} 1 & t_2 & \ldots & t_s \\ p_1 t_1 & p_1 & \ldots & p_s \\ \vdots & & & \\ p_1^{s-1} & p_2^{s-1} & \ldots & p_s^{s-1} \end{pmatrix}$$

where $p_i$ are real positive numbers and $t_j$ are functions. Based on the results we derive an approximation of the N-soliton solution of KdV as the spatial derivative of $\sum_{j=1}^{N} p_j \tanh(f_j - \text{atanh} \sum_{l=1,k \neq j}^{N} b_{j,l} k t_s)$, where $b_{j,k}$ are real numbers and in the case of two solition solution the approximate solution is identical to the exact one. Error analysis of the approximations is also discussed.
PRIMAL-DUAL WEAK GALERKIN FINITE ELEMENT METHODS

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Weak Galerkin (WG) is a newly developed finite element technique for partial differential equations (PDEs) where the differential operators (e.g., gradient, divergence, curl, Laplacian, etc.) in the variational forms are reconstructed by using a framework in the theory of distributions. The framework yields discrete weak differential operators such as weak gradients and weak curls. The computation of the discrete differential operators involves the solution of inexpensive problems defined locally on each element. Appropriately defined stabilizers are employed to provide weak continuities that are necessary for the original formulations. The fundamental difference between the weak Galerkin finite element method and other existing finite element methods (FEM) is the use of weak functions and weak derivatives (i.e., locally reconstructed differential operators) in the design of numerical schemes based on conventional weak forms for the underlying PDE problems. Weak Galerkin is a natural extension of the classical conforming and non-conforming finite element method and has advantages over FEM in many aspects. Due to its great structural flexibility, the weak Galerkin finite element method is well suited to most PDEs by providing the needed stability and accuracy in approximations. In this talk, the speaker will introduce the basic ideas and a general framework for WG methods by using the second order elliptic equation as a model problem. The speaker will then discuss a recent development of WG, known as “Primal-Dual Weak Galerkin (PD-WG)”. The essential idea of PD-WG is to interpret the numerical solutions as constrained minimization of some functionals with constraints that mimic the weak formulation of the PDEs by using the weak derivatives. The resulting Euler-Lagrange equation offers a symmetric finite element scheme involving both the primal and the dual variable (also known as the Lagrange multiplier). The primal-dual weak Galerkin methods will be applied to several challenging problems for which existing methods have difficulty in applying; these problems include the second order elliptic equations in nondivergence form, Fokker-Planck equation, first order convection equations, and elliptic Cauchy problems. Finally, the speaker will introduce an abstract framework for the PD-WG method and discuss its great potential in other scientific applications.

ON GLOBAL EXISTENCE FOR MASS-SUPERCritical NONLINEAR FRACTIONAL HARTREE EQUATIONS

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In this talk, I will introduce some results on dynamics of time-dependent fractional Hartree equations, that is, the nonlinear fractional Schrodinger equations with Hartree type nonlinearity. In mass-supercritical and energy-subcritical case, we establish a threshold condition by sharp Hardy-Littlewood-Sobolev inequality and the Pohozaev identity. Under certain conditions, our results leads to a global existence of solutions to the Cauchy problems in energy space.
A NEW INTEGRAL TRANSFORM APPROACH TO SOLVING HYPERBOLIC EQUATIONS IN THE CURVED SPACE-TIMES

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In this talk we will present the integral transform that allows us to construct solutions of the hyperbolic partial differential equation with variable coefficients via solutions of a simpler equation. This transform was used to investigate several well-known equations such as generalized Tricomi equation, the Klein-Gordon equation of the quantum field theory in the de Sitter and Einstein-de Sitter space-times of the expanding universe. In particular it was found a value of mass that makes the Klein-Gordon equation huygensian. A special attention will be also given to the global in time existence of self-interacting scalar field in the de Sitter universe and to the Higuchi bound of the quantum field theory and to equations with the Higgs potential.

OPTIMAL STOPPING PROBLEM IN ASSET REPLACEMENT UNDER IMPROVING TECHNOLOGY

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The talk derives a homogeneous PDE, which arises in the optimal replacement of capital assets (equipment, machines) under evolving uncertain costs, and solves it analytically in an important special case. The asset replacement decision as a stopping problem has been considered under uncertain age-dependent costs without technological change in (Ye 1990, Mauer and Ott 1995, Dobbs 2004) and others. This popular approach to asset replacement uses dynamic programming and the real options concept (Dixit and Pindyck 1994). It introduces the value of the real option to exchange the current asset with a new one as an optimal stopping problem to find the threshold replacement cost when the replacement should occur. Next, the Bellman equation for the stopping problem is formulated and solved. Because of technical complexity of the Bellman equation, the stopping problem is usually restricted to linear and geometric age-dependent cost. Here we extend this technique to the case of uncertain age- and time-dependent costs under proportional technological change.